Fig. S1. The 2D spatial correlation characterizing lane formation. (A) Spatial correlation \( C(x_t, x_n) \) at radius of rotation \( R = 8\sigma \), defined as \( \langle s(0, 0) \cdot s(x_t, x_n) \rangle \) in the particle reference frame, where \((x_t, x_n)\) stands for coordinates tangential and normal to the particle orientation. Particles exhibit strong correlations only along the particle orientation, indicating lane formation. In fact, the formed lanes lie themselves in a small angle lagging behind the particle orientation, as the reorientation of the lanes generally requires multiple particle collisions and reorganization. (B) Dependence of corresponding correlation length \( \xi \) on \( R \). Correlation lengths are evaluated along (red) and perpendicular (blue) to the lane orientation. They are denoted as the distance between the origin and the position where the correlation \( C \) becomes less than 0.1. As \( R \) increases, lanes become longer but remain narrow. Note that \( \xi \) is the correlation length of the local order parameter, rather than that of the order-parameter fluctuations discussed in Results. The latter diverges at the critical point \( R_c \).

Fig. S2. Kurtosis, defined as \( \langle (\Delta x^4) / (\Delta x^2)^2 \rangle \), of the distribution of stroboscopic displacement \( \Delta x \) along the \( x \) axis (shown in Fig. 1C) at various radii of rotation \( R \). The kurtosis, which has a lower bound of 1, is close to its Gaussian value 3 for all \( R \) investigated, indicating that particle displacement follows a normal distribution. Minor suppression of the kurtosis below 3 occurs when \( R \) is decreased below \( 4\sigma \), because then the number of collisions per orbit becomes too small to generate large displacements. Ultimately, the kurtosis increases again due to competing effects driven by the presence of a critical point just below \( R = 2\sigma \).
Fig. S3. Dynamics of spatial correlation function $C(r)$ characterizing domain coarsening in experiment. Spatial correlation function $C(r)$, defined as $\langle s(0) \cdot s(r) \rangle$, calculated at different times $t = 25$ s (black), 100 s (red), 250 s (blue) and 500 s (purple), corresponding to data presented in Fig. 5C. The correlation length $\xi$ can be approximated as the distance $r$ where $C(r)$ first passes zero, and its time evolution is shown in Fig. 5C. (Inset) by normalizing the distance $r$ by the time-dependent correlation length $\xi(t)$, spatial correlation functions at different time collapse, indicating that the system evolves in a self-similar manner over time.

Fig. S4. Distribution of the local order parameter $s = \phi_A - \phi_B$ measured in subcells of side length $L = 12.5\sigma$, at different $R = 1.2\sigma, 1.4\sigma, 1.6\sigma, 1.8\sigma, 2.0\sigma, 2.2\sigma, 2.4\sigma, 3.0\sigma$ (color: from blue to red), at symmetric composition. As $R$ decreases, the order-parameter distribution changes from a Gaussian distribution centered at 0 to a symmetric bimodal distribution, indicating a phase transition and associated spontaneous symmetry breaking in the order parameter. The absolute value of the peak position denotes the global order parameter $M$ of the system.

Single-particle behavior
(real time)

Movie S1. Single-particle behavior. The opaque hemisphere of the Janus colloids (diameter $\sigma = 3\mu m$) corresponds to the metal-coated side; the transparent hemisphere corresponds to the silica side. At the start of the video, a static magnetic field in the vertical direction is present. Upon applying an electric field perpendicular to the image plane, particles differentiate into two types that swim in opposite directions. In the second half of the video, a rotating magnetic field is used instead, causing particles to move in circular orbits. Particles of the opposite type collide with each other when nearby.

Movie S1
Lane formation at large $R$
(real time)

$\rho = 0.44, R = 36\sigma, \phi_A = 0.5$

Movie S2. Lane formation at large $R$. This video shows lane formation of Janus colloids when orbiting at a very large radius $R = 36\sigma$ in a system with symmetric composition. (Inset) The two particle species are color-coded. The video plays in real time. The viewing window is $103 \times 77 \mu m^2$.

Phase separation at small $R$
(real time)

$\rho = 0.45, R = 0.66\sigma, \phi_A = 0.5$

Movie S3. Phase separation at small $R$. This video shows the zoomed-in view of the phase separation dynamics for this driven binary colloidal system at a small radius of circular motion $R = 0.66\sigma$ and symmetric composition. The video plays in real time. The viewing window is $165 \times 124 \mu m^2$.

Critical behavior in simulation
(sped up $10^5$x)

$\rho = 0.37, R = 1.84\sigma, \phi_A = 0.5$

Movie S4. Critical behavior in simulation. This video shows behavior of Janus colloids at the critical radius of circular motion $R = 1.84\sigma$ and symmetric composition. Color-coding is based on the local order parameter $s(r)$, varying from blue to red as the value increases. The video plays $10^5$x faster than real time. The viewing window is $3,600 \times 3,600 \mu m^2$. 

Movie S4
Spinodal decomposition  
(sped up 32x)

\[ \rho = 0.42, \ R = 0.88\sigma, \ \phi_A = 0.5 \]

**Movie S5.** Spinodal decomposition. This video shows spinodal decomposition of Janus colloids at a small radius of circular motion \( R = 0.88\sigma \) and symmetric composition. The video plays 32x faster than real time. The viewing window is 975 \( \times \) 743 \( \mu m^2 \).

**Movie S5**

Nucleation and growth  
(sped up 30x)

\[ \rho = 0.36, \ R = 0.5\sigma, \ \phi_A = 0.15 \]

**Movie S6.** Nucleation and growth. This video shows nucleation and growth of Janus colloids at a small radius of circular motion \( R = 0.5\sigma \) and asymmetric composition \( \phi_A = 0.15 \). The video plays 30x faster than real time. The viewing window is 720 \( \times \) 720 \( \mu m^2 \).

**Movie S6**

Nuclei formation  
(sped up 5x)

\[ \rho = 0.36, \ R = 0.5\sigma, \ \phi_A = 0.15 \]

**Movie S7.** Nuclei formation. This video shows nuclei formation of Janus colloids at a small radius of circular motion \( R = 0.5\sigma \) and asymmetric composition \( \phi_A = 0.15 \). Nascent nuclei have irregular shapes and fluctuate wildly. They merge and grow with time, with sharpening interface and increasingly spherical shape. The video plays 5x faster than real time. The viewing window is 144 \( \times \) 144 \( \mu m^2 \).

**Movie S7**
Spreading of a droplet
(spnd up 20x)

$\rho = 0.42, \ R = 0.88\sigma, \ \phi_A = 0.5$

Movie S8. Spreading of a droplet. This video shows spreading of a droplet of Janus colloids onto a larger domain at a small radius of circular motion $R = 0.88\sigma$ and symmetric composition. The video plays 20x faster than real time. The viewing window is $144 \times 144 \ \mu m^2$. 

Movie S8